Engineering Mechanics

Dynamics

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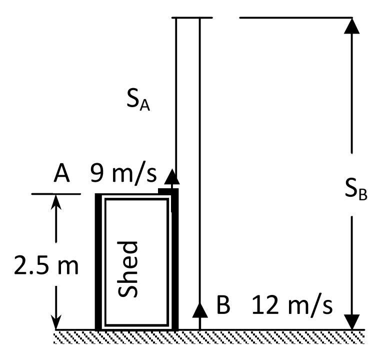
Engineering Mechanics-Dynamics

Solutions for
University Question Paper-2010

1.Boy 'A 'thrown a ball vertically up with a speed of 9 m/sec from the top of a shed of 2.5m high . Boy 'B" on the ground at the same instant throws a ball vertically up with a speed of 12 m/sec . Determine the time at which the two balls will be the same height above the ground. What is the height?

<u>Boy A: Consider the upward motion of the ball</u>



u = 9 m/ Sec
a = - g = - 9.81 m/ sec^2
t = time taken by the ball to reach the height "h"
S_A = Height reached by the ball A

Equation of motion
$s = ut + \frac{1}{2}at^2$
$S_A = 9\,t + ½ (- 9.81) \times t^2$
$S_A = 9\,t - 4.905\,t^2$ …………(1)

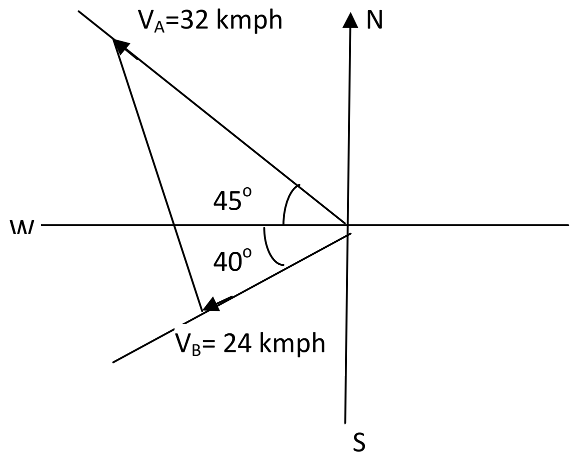
<u>Boy B : Consider the upward motion of the ball</u>
u = 12 m/ sec
a = - g = - 9.81 m/ sec^2
S_B = Height reached by the ball B

$S_B = S_A + 2.5$
t = time taken (time remains same for both the cases)

Equation of motion
$S = ut + ½\, at^2$
$S_B = 12\,t - ½ \times 9.81\,t^2 = 12t - 4.905\,t^2$ …………. (2)

Substitute (1) in (2)
$S_B = S_A + 2.5 = 9\,t - 4.905\,t^2 + 2.5 = 12\,t - 4.905\,t^2$
$12t - 9\,t = 2.5$
t = 0.833 sec.
There fore
$S_A = 9 \times 0.833 - 4.905 \times 0.833^2 = 4.093$ m
$S_B = 4.093 + 2.5 = 6.593$ m
Height of the two ball from ground = 6.593 m (Ans)

2. Two ships leave a port at the same time. First ship steams North–West at 32 km/hr and the second 40° south of West at 24 km/ hr. What is the speed of second ship relative to the first ship in km/ hr? Also find the time at which the ship will be 160 km apart.



Velocity of ship B with respect to Ship A ($V_{B/A}$) =

$$\sqrt{V_A^2 + V_B^2 - 2V_AV_B\cos(45+40)}$$

$$\sqrt{32^2 + 24^2 - 2 \text{ x } 32 \text{ x } 24 \cos 85}$$

$V_{B/A} = \sqrt{1461.12}$ = 38.29 kmph

Time at which the ships are 160 km away

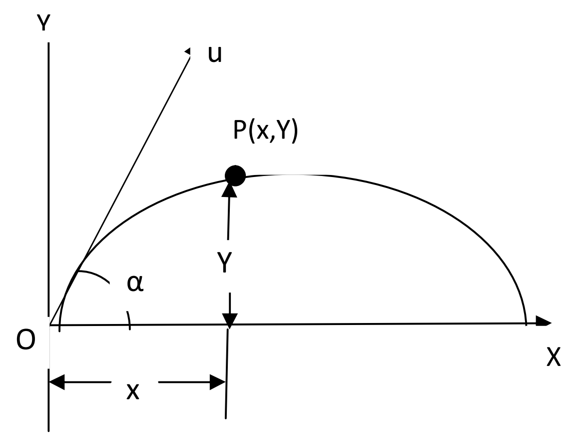
Relative distance = Relative velocity x time

160 = $V_{B/A}$ x t

t = $\frac{160}{38.29}$ = 4.178 hrs

Unit II

3.a) Derive the equation for the path of a projectile b) Derive expressions for time of flight, maximum height attained, horizontal range of the projectile.



Consider a particle projected from a point 'O' at an angle α From the horizontal. Let u – be the velocity of projection Consider any point P (x,y) as the position of particle after time 't' sec with x and y co- ordinates the horizontal

The horizontal component $u_x = u\cos\alpha$

The vertical component $u_y = u\sin\alpha$

Vertical distance traveled after time t = y

Using the relation S = ut + $\frac{1}{2}$ at^2

We can get y = (usinα) t – $\frac{1}{2}$ gt^2

Horizontal distance moved after time t = x

using the reletion S = v x t (since horizontal component of motion is constant)

we can get x = ucos α . t

$t = \frac{x}{u cos\propto}$

Subsisting the equation 2 in 1

$$Y = u \sin \propto x \frac{x}{u \cos\propto} - \frac{1}{2} g \frac{x^2}{u^2 cos^2\propto}$$

Equation of projectile $y = x \tan \propto - \frac{gx^2}{2\, u^2 cos^2\propto}$

<u>3 b) Derive expressions for time of flight, maximum height attained, horizontal range of the projectile</u>.

<u>Time of flight of a projectile on a horizontal plane</u>

We know that at the end of a projectile (ie ., after reaching the ground) the displacement is zero

$Y = u \sin \alpha * t - \frac{1}{2} g t^2$

$0 = u \sin \alpha * t - \frac{1}{2} g t^2$

$u \sin \propto * t = \frac{1}{2} g t^2$

$t = \frac{2usin \propto}{g}$

<u>Maximum height of a projectile on a horizontal plane</u>

In a vertical direction the initial velocity $u_y = u \sin \propto$

Final velocity $u_y = 0$

Let H – Max height reacted by the projectile

From the equation $v^2 - u^2 = -2gs$

we can write $v_y{}^2 = u_y{}^2 - 2gH$

There fore we get $0 = (u \sin \propto)^2 - 2gH$

$2gH = u^2 sin^2 \propto$

$H = \frac{u^2 sin^2 \propto}{2g}$

<u>Horizontal range of the projectile</u>

Horizontal component of velocity of projection

$u_x = u \cos \propto$

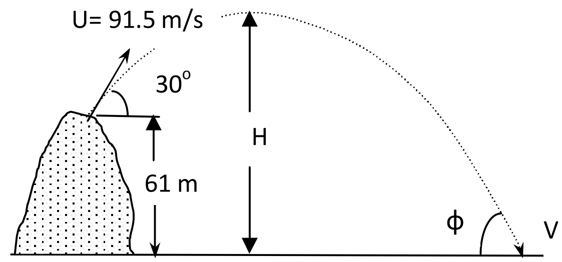
We know that $t = \frac{2u \sin\propto}{g}$

Range (R) = Horizontal Velocity x time $= u \cos \propto \times \frac{2u \sin\propto}{g}$

$$R = \frac{u^2 \sin 2 \propto}{g} \quad [\, since \sin 2 \propto = 2 \sin \propto \cos \propto]$$

4. A soldier fires a bullet at an angle of 30°(upward from the horizontal) from his position on hill to strike a target which is 61 m lower than the position of the soldier . The initial velocity of the bullet is 91.5 m/ sec . Calculate (i) The maximum height to which the bullet will raise above the horizontal (ii) the actual velocity which it will hit the target (iii) The total time required for the flight of the bullet.

<u>(i) Maximum height reached by the bullet</u>



u = 91.5 m/ sec α = 30°

$H = \frac{u^2 sin^2 \propto}{2g} = \frac{91.5^2 \times sin^2 30}{2\times 9.81} = 106.68\ m$

H =106.68 m

(ii) Actual velocity to hit the target

$u_y = u \sin \propto = 91.5 \times \sin 30 = 45.75\ m/\ sec$

a = -9.81 m/ sec^2; S = - 61 m ; t = time taken to hit the target

$s = ut + \frac{1}{2} at^2$

$-61 = 45.75 \times t - \frac{1}{2} \times 9.81 \times t^2$

$4.905t^2 - 45.75t - 61 = 0$

$t = \frac{45.75 \pm \sqrt{45.75^2 + 4\times 4.705 \times 61}}{2\times 4.705}$; t = 11.124 sec.

Time required to hit the target =11.124 sec

Horizontal distance $s = u \cos \propto \times$ t

$= 91.5 \times \cos 30 \times 11.124 \quad = 881.48\text{m}$

Horizontal Component Velocity $v_x = u \cos \propto$

$= 91.5 \times \cos 30 = 79.24\ m/\ sec$

Using the relation $v^2 - u^2 = 2as$

$v_y{}^2 = -2 \times 9.81 \times (-61) + 45.75^2$

$v_y = 57.35\ m/sec$

Velocity to hit the target $v = \sqrt{v_x{}^2 + v_y{}^2}$

$= \sqrt{79.241^3 + 57.35^2}$

= 97.81 m/ sec

Unit- III

5. A car of mass 1,500 kg uniformly accelerated. Its speed increased from 50 kmph to 75 kmph after travelling a distance of 200 m. The resistance of the motion of the car is 0.2 % of the weight of the car. Determine: (i) The maximum power required. (ii) The power required to maintain constant speed of 75 kmph.

m = 1500 kg u= 50 km/hr = 13.88 m/sec

s-200m v= 75 km/hr= 20.83 m/sec

Using the relation

$$v^2 - u^2 = 2as$$
$$20.83^2 - 13.88^2 = 2 \times a \times 200$$
$$a = 0.603\ m/sec^2$$

Frictional resistance = 0.2% of wt. of the car

$$= \frac{0.2}{100} \times 1500 \times 9.81 = 29.43\ N$$

Using D' Alembert's Principle

$$\sum F = m \times a$$
$$F_t - F_r = m \times a$$
$$F_t - 29.43 = 1500 \times 0.603$$
$$F_t = 933.93N.$$

(i) Max .Power required $= F_t \times v$

= 933.93 x 20.83

= 19.453 kW

(ii) The power required to maintain constant speed of 75 kmph

V= 75 kmph = 20.83 m/se. $when\ speed\ in\ constant\ a = 0$

$\therefore \sum F = m \times a$

$\therefore F_t - F_r = 0$

$F_t = F_r = 29.43\ N$

$Power\ required = F_t \times v = 29.43 \times 20.83 = 0.613\ kW$

6. Two blocks of A and B of weight 80 N and 60 N are connected by a string passing through a smooth pulley as shown in fig. 1. Calculate the acceleration of body and the tension in the string

$W_A = 80\ N$

$W_B = 60N$

Let T be the tension on the string on both sides

a = acceleration of both the blocks

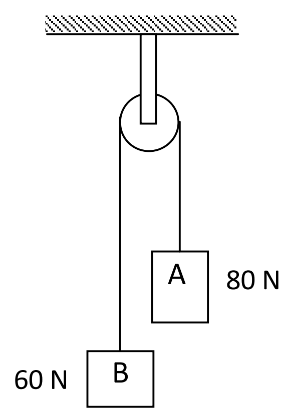


Fig.1

Consider down ward motion of block 'A'

D' Alambert 's Principle

$$\sum F = m \times a$$

$$W_A - T = m_A \times a$$

$$80 - T = \frac{80}{9.81} \times a$$

$$80 - T = 8.15\,a \qquad \cdots\cdots \quad (1)$$

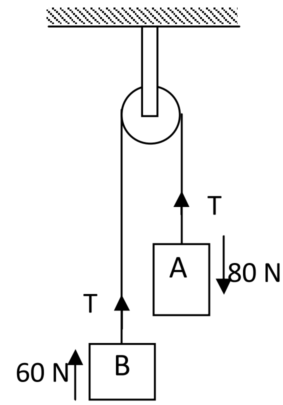


Fig.1

Consider upward motion of block'B'

D' Alambert 's Principle

$$\sum F = m \times a$$

$$T - W_B = m_B \times a$$

$$T - 60 = \frac{80}{9.81} \times a$$

$$T - 60 = 6.12\,a \qquad \cdots\cdots \quad (2)$$

Add (1) & (2)

$$14.27a = 20$$

$$a = 1.40 \ m/sec^2$$

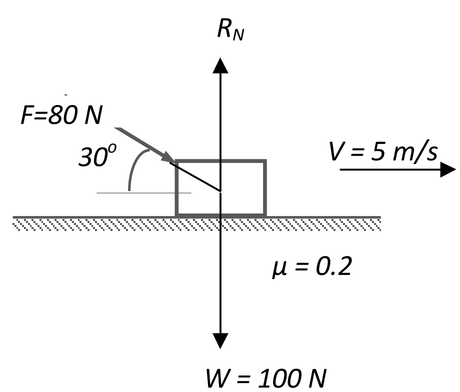
Substitute the value of a in equation (2)

$$T = 6.12 \times 1.4 + 60 = 68.56\,N$$

UNIT – IV

7. A block weighing 100 N is moving along a horizontal rough surface of friction co-efficient 0.2 with a velocity of 5 m/s. A push of 80 N inclined at 30⁰ to the horizontal acts on the block. Using work – energy principle, find the velocity of the block after it had moved through a distance of 20 m.

Self wt W is acting vertically downwards
Normal Reaction R_N is acting vertically upwards
Force 80N act 30º to horizontal
Vertical component $F_Y = 80 \sin 30$
Horizontal component $F_X = 80 \cos 30$



Resolving the force perpendicular to the plate

$$R_N = W + F_y = 100 + 80 \sin 30 = 140\,N$$

$F_r = \mu \times R_N \ \ = 0.2 \times 140 \ = 28\ N$

By Work–Energy Relation ship

$$\sum F \times s = \frac{1}{2} m({v_2}^2 - {v_1}^2)$$

$$(F_x - F_r) \times s = \frac{1}{2} m({v_2}^2 - {v_1}^2)$$

$$(80 \times \cos 30 - 28) \times 20 = \frac{1}{2} \times \frac{100}{9.81} \times ({v_2}^2 - 5^2)$$

$${v_2}^2 - 25 = 161.99$$

$$\boldsymbol{v_2 = 13.67\ m/sec}$$

8. A spring of stiffness 0.5 N/mm is placed, horizontally and a ball of mass 5 kg strikes the spring horizontally with a velocity equal to that attained by a vertical fall of height 120 mm. Find the maximum compression of the spring using law conservation of mechanical energy.

Stiffness k= 0.5 N/ mm = 500 N/m ; m= 5 kg

Velocity of the ball when striking the spring = velocity of ball falling from at height of 120 mm

$$= \sqrt{2 \times g \times 0.12}$$

$$v_1 = 1.53\ m/sec$$

By work energy relationships

Total Work done = Change in Kinetic energy

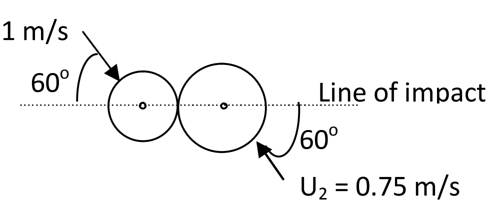
$$-\frac{1}{2} kx^2 = \frac{1}{2} m({v_2}^2 - {v_1}^2)$$

$$-\frac{1}{2} \times 500 \times x^2 = \frac{1}{2} \times 5(0^2 - 1.53^2)$$

$$x = 0.153m$$

UNIT – V

9. A ball of mass 500 grams, moving with a velocity of 1 m/s impinges on a ball of mass 1 kg, moving with a velocity of 0.75 m/s. At the time of impact the velocities of the balls are parallel and inclined at 60⁰ to the line joining their centers. Determine the velocities and direction of the balls after impact. Take e =0.6.



m_1 = 500gms = 0.5kg $u_1 = 1\ m/sec$

m_2 = 1kg $u_2 = 0.75\ m/sec$

$\alpha_1 = 60°$ $\alpha_2 = 60°$

e = 0.6

Considering horizontal component of velocities along the line of impact and

Applying Law of conservation of momentum

$m_1 u_1 \cos \propto_1 + m_2 u_2 \cos \propto_2 = m_1 v_1 \cos \theta_2 + m_2 v_2 \cos \theta_2$

$(0.5 \times 1 \times \cos 60) + (1 \times (-0.75) \cos 60) = (0.5 \times v_1 \cos \theta_1) + (1 \times v_2 \cos \theta_2)$

$0.5 \times v_1 \cos \theta_1 + v_2 \cos \theta_2 = 0.25 - 0.375$

$0.5\, v_1 \cos \theta_1 + v_2 \cos \theta_2 = -0.125 \quad \ldots\ldots\ldots\ldots\ldots\ldots. \quad (1)$

Newton's law of collision

$v_2 \cos \theta_2 - v_1 \cos \theta_1 = e(u_1 \cos \alpha_1 - u_2 \cos \alpha_2)$

$v_2 \cos \theta_2 - v_1 \cos \theta_1 = 0.6(1 \times \cos 60 — 0.75 \cos 60)$

$v_2 \cos \theta_2 - v_1 \cos \theta_1 = 0.525$ --→ (2)

The component normal to line of impact

$u_1 \sin \propto_1 = v_1 \sin \theta_1$

$1 \sin 60° = v_1 \sin \theta_1$

$v_1 \sin \theta_1 = 0.866$ −−−−−−−−−−−−−−−−−→ (3)

$u_2 \sin \propto_2 = v_2 \sin \theta_2$

$0.75 \sin 60° = v_2 \sin \theta_2$

$v_2 \sin \theta_2 = 0.6495$ −−−−−−−−−−−−−−−−−→ (4)

$(1) - (2) \Rightarrow \quad 1.5 v_1 \cos \theta_1 = -0.65$

$v_1 \cos \theta_1 = -\ 0.4333$ ……..(5)

Substituting this in eqn. (2)

$v_2 \cos \theta_2 = 0.09166$ ……..(5)

We know that

$v_1 = \sqrt{(v_1 sin\theta 1)^2 + (v_1 cos\theta 1)^2} \quad ; \quad v_1 = \sqrt{(0.866)^2 + (-0.433)^2}$

$v_1 = 0.968$ m/s

$\tan \theta_1 = v_1 \sin \theta_1 / v_1 \cos \theta_1$

$= 0.866/-0.433 = -2$

$\theta_1 = -63.43^o$

$v_2 = \sqrt{(v_2 sin\theta 2)^2 + (v_2 cos\theta 2)^2} \quad ; \quad v_2 = \sqrt{(0.6495)^2 + (0.0916)^2}$

$v_2 = 0.655$ m/s

$\tan \theta_2 = v_2 \sin \theta_2 / v_2 \cos \theta_2$

$\tan \theta_2 = 0.6495 / 0.0916 = 7.09$

$\theta_2 = 81.96°$

$v_1 = 0.968\, m/sec \quad v_2 = 0.655\ m/sec \quad \theta_1 = -63.4° \quad \theta_2 = 81.96°$

10. Two bodies one of which is 200 N with a velocity of 10 m/s and the other of 100 N with a velocity of 10 m/s move towards each other and impinges centrally. Find the velocity of each body after impact if the co-efficient of restitution is 0.6. Find also, the loss in kinetic energy due to impact.

$W_1 = 200N \qquad u_1 = 10m/sec \qquad e = 0.6$
$W_2 = 100N \qquad u_2 = 10m/sec$

Law of conservation of momentum
$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

$$\frac{200}{9.81} \times 10 + \frac{100}{9.81} \times (-10) = \frac{200}{9.81} \times \ v_1 + \frac{100}{9.81} \times v_2$$

$200v_1 + 100v_2 = 2000 - 1000$
$200v_1 + 100v_2 = 1000$
$2v_1 + v_2 = 10 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$

Newton's law of collision.

$(v_2 - v_1) = e(u_1 - u_2)$
$(v_2 - v_1) = 0.6[10 - (-10)]$
$(v_2 - v_1) = 12$………………..(2)

$(1) - (2) \cdots\cdots \rightarrow 3v_1 = \ -2$

$$v_1 = \ -0.6666m/sec$$

Apply v_1 value in equation (1)

$2 \times (-0.666) + v_2 = 10$
$v_2 = 11.332\ m/sec$

Loss of kinetic energy = $= \frac{1}{2}(m_1{u_1}^2 + m_2{u_2}^2) - \frac{1}{2}(m_1{u_1}^2 + m_1{v_2}^2)$

$$= \frac{1}{2}(\frac{200}{9.81} \times 10^2 + \frac{100}{9.81} \times 10^2\ ) - \frac{1}{2}(\frac{200}{9.81} \times (-0.666)^2 + \frac{100}{9.81} \times (11.332)^2\ )$$

$= 1529.05 - 1315.28$
$= 213.76\ N - m$

Engineering Mechanics-Dynamics

Solutions for
University Question Paper-2011

UNIT – I

1. The equation of motion of a particle is given, acceleration (a) in terms of time (t) as below. $a = 3t^2 + 2t + 4$, in which acceleration is in m/s^2 and time "t" is in seconds. It is observed that the velocity of the particle is 12 m/s after 4 seconds; and the displacement of the particle is 8 m after 4 seconds. Determine, (i) Velocity after 8 seconds. (ii) Displacement after 2 seconds.

$$a = 3t^2 + 2t + 4 ------\rightarrow (1)$$

$$\frac{dv}{dt} = 3t^2 + 2t + 4; \qquad dv = (3t^2 + 2t + 4)\, dt$$

Integrating both sides.

$$\int dv = \int (3t^2 + 2t + 4)\, dt$$

$$v = \frac{3t^3}{3} + \frac{2t^2}{2} + 4t + c_1$$

$$v = t^3 + t^2 + 4t + c_1 -----\rightarrow (2)$$

When t = 4sec ; v = 12m/sec ; apply on eq (2)

$$12 = 4^3 + 4^2 + (4 \times 4) + c_1$$

$$c_1 = 12 - 96 = -84$$

$$\therefore (2) \rightarrow v = t^3 + t^2 + 4t - 84\, ; ---\rightarrow (3)$$

$$\frac{ds}{dt} = t^3 + t^2 + 4t - 84$$

$$ds = (t^3 + t^2 + 4t - 84)\, dt$$

Integrating both sides

$$s = \frac{t^4}{4} + \frac{t^3}{3} + \frac{4t^2}{2} - 84t + c_2$$

$s = \frac{t^4}{4} + \frac{t^3}{3} + 2t^2 - 84t + c_2 - - - \rightarrow (4)$

When t = 4 sec ; $s = 8m;$ apply on eq → (4)

$8 = \frac{4^4}{4} + \frac{4^3}{3} + (2 \times 4^2) - (84 \times 4) + c_2 - - - \rightarrow (4)$

c_2=226.67

$\therefore$ (3) → $s = \frac{t^4}{4} + \frac{t^3}{3} + 2t^2 - 84t + 226.67 - - - \rightarrow (5)$

When t =sec; $v = ?$

Substitute t value in eq (4)

$v = 8^3 + 8^2 + (4 \times 8) - 84$

$v = 524m/sec$

When t =2sec; $s = ?$

Substitute t value in eq (5)

$s = \frac{2^4}{4} + \frac{2^3}{3} + (2 \times 2^2) - (84 \times 2) + 226.67$

$s = 73.337m$

2. Two ships leave a port at the same time. First ship steams North–West at 32 km/hr and the second 40^0 South of West at 24 km/hr. What is the speed of the second ship relative to the first ship in km/hr? Also, find the time at which the ships will be 160 km apart?

Refer 2010 Q.P

UNIT – II

3. Two balls are projected from the same point in directions inclined at 60^0 and 30^0 to the horizontal. If they attain the same maximum height, what is the ratio of their velocities of projection?

$\propto_1 = 60°$; $\propto_2 = 30°$

H- Maximum height of their Projectile

For $\propto = 60°$; $H = \frac{{u_1}^2 sin^2 60}{2g}$ ————————————→ (1)

For $\propto = 30°$; $H = \frac{{u_2}^2 sin^2 30}{2g}$ ————————————→ (2)

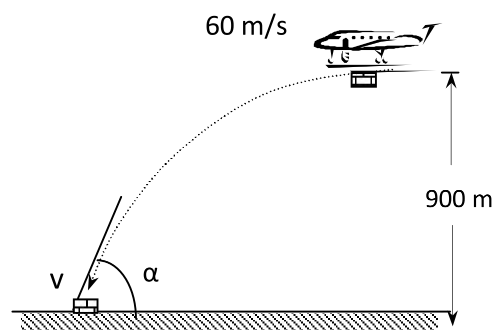
Equate− − −(1)&(2)

$$\frac{{u_1}^2 sin^2 60}{2g} = \frac{{u_2}^2 sin^2 30}{2g}$$

$$\frac{{u_1}^2}{{u_2}^2} = \frac{sin^2 30}{sin^2 60}; \quad \frac{u_1}{u_2} = \sqrt{\frac{0.25}{0.75}}$$

$$\frac{u_1}{u_2} = 0.577$$

4. An aero plane is flying horizontally with a constant speed of 60 m/s, at an altitude of 900 m. If the pilot drops a package with same horizontal speed of 60 m/s, determine the velocity when the package hits the ground and its angle with horizontal.



Horizontal velocity of dropping Package = Velocity of Aircraft

$v_x = 60 m/sec$

Vertical velocity = 0

Considering down ward motion of package

$u = 0$; $a = g$; $s = 900$ m

$$s = ut + \frac{1}{2}at^2$$

$$900 = 0 + \frac{1}{2} 9.81 \times t^2$$

$$t = \sqrt{\frac{900}{4.905}} = 13.54 \text{ sec}$$

$v = u + at = 0 + 9.81 \times 13.54$

$v_y = 132.82$ m/s

Velocity of the package hit the ground $v = \sqrt{{v_x}^2 + {v_y}^2}$

$$= \sqrt{132.82^2 + 60^2} \qquad v = 145.75 m/sec$$

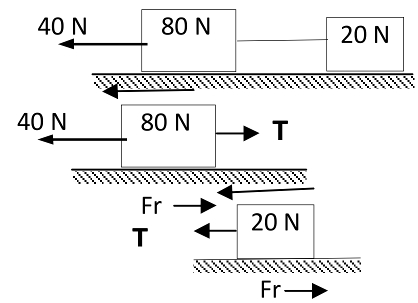
Angle with horizontal $\quad \propto = \tan^{-1}\frac{v_y}{v_x}$

$= \tan^{-1}\frac{132.82}{60}$

$\propto \ = \mathbf{65.68^\circ}$

UNIT – III

5. Two weights 80 N and 20 N are connected by a thread and move along a rough horizontal plane under the action of force 40 N, applied to the first weight of 80 N as shown below. The co-efficient of friction between the sliding surfaces of the weights and the plane is 0.3. Determine the acceleration of the weights and the tension in the thread using D' Alembert's principle.



Consider 80N weight body

Frictional force $F_r = \mu \times R_N$

$= \mu \times w \ = \ 0.3 \times 80$

$= 24N$

By D' Alembert Principle

$\sum F = m\, a$

$F - T - F_r \ = m\, a$

$40 - T - 24 = \dfrac{80}{9.81} \times a$

$16 - T = 8.15\, a - -\!\!-\!\!\rightarrow \ (1)$

Consider 20N weight body

$F_r = \mu \times R_N = \mu \times w \quad = 0.3 \times 20 = 6N$

By D' Alembert Principle

$\sum F = m \times a$

$T - F_r \ = m \times a$

$T - 6 = \dfrac{20}{9.81} \times a$

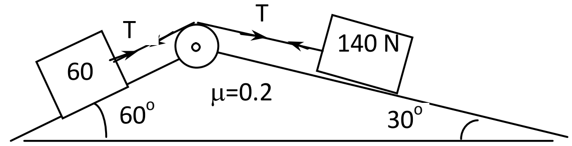
$T - 6 = 2.03\, a - -\!\!-\!\!\rightarrow \ (2)$

$(1) + (2) \rightarrow 10 = 10.18\, a$

$a = 0.98 m/sec$

Substitute a value in eq (2) $\quad$ T = $\ 2.03 \times 0.98 + 6 \qquad = 7.99N$

6. The following figure shows two blocks of weight 60 N and 140 N placed on two inclined surfaces and connected by an inextensible string. Calculate the acceleration of the system and the tension in the string. Take μ = 0.2.



Assuming the mass 140 N is moving down & 60 N mass is moving up

Consider upward motion of 60N block

Frictional force $F_r = \mu \times R_N$

$$= \mu \times w\cos 60 \;=\; 0.2 \times 60\cos 60 \;=\; 6N$$

By D' Alembert Principle

$$\sum F = m \times a$$

$$T - F_r = w\sin\propto = m \times a$$

$$T - 6 - 60\sin 60 = \frac{60}{9.81} \times a$$

$$T - 57.96 = 6.116\,a - --\rightarrow\ (1)$$

Consider down ward motion of 140 N block

Frictional force $F_r = \mu \times R_N$

$$= \mu \times w\cos 30 \;=\; 0.2 \times 140\cos 30 \;=\; 24.248N$$

By D' Alembert Principle

$$\sum F = m \times a$$

$$w\sin\propto - T - F_r = m \times a$$

$$140\sin 30 - T - 24.248 = \frac{140}{9.81} \times a$$

$$45.75 - T = 14.27\,a - --\rightarrow\ (2)$$

Solving 1 & 2 we get a = -0.598 m/s and T = 61.62 N

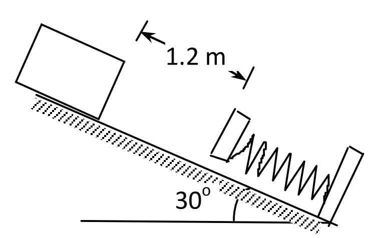
8. A block of mass 75 kg slides down a 30° inclined plane from rest, as shown below. After moving 1.2 m, the block strikes a spring whose modulus is 20 N/mm, Determine the maximum deformation of the spring. Take the coefficient of kinetic friction between the block and the plane is 0.25.

m=75 kg α = 30° k =20 N/mm = 20,000 N/m $\mu = 0.25$

$x = deformation\ of\ spring$

Work –Energy relationship

W.D by block + W.D by Spring = Change in kinetic Energy



$$\sum F \times (s + x) - \frac{1}{2}kx^2 = 0$$

$$(w \sin \propto - F_r)(s + x) - \frac{1}{2}kx^2 = 0$$

$$(75 \times 9.81 \sin 30 - 0.25 \times 75 \times 9.81 \cos 30)(1.2 + x) - \frac{1}{2} \times 20{,}000 \times x^2 = 0$$

$$(367.875 - 159.29)(1.2 + x) - 10{,}000x^2 = 0$$

$$10000x^2 - 208.58x - 250.296 = 0$$

$$x = \frac{208.58 \pm \sqrt{208.58^2 - 4 \times 10{,}000 \times 250.296}}{2 \times 10{,}000}$$

$$= \frac{208.58 \pm 3171.01}{20{,}000}$$

$$x = 0.168m$$

UNIT - V

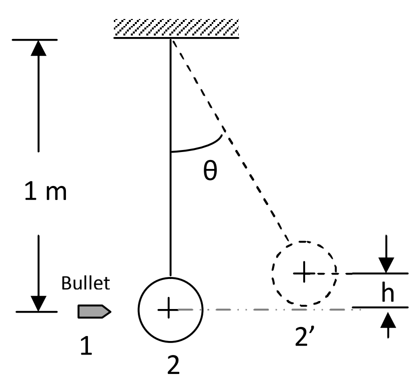
9. A bullet of mass 25 grams is moving with a velocity of 500 m/ s and fired into a body of 12 kg, which is suspended by a string, fixed at top, 1 m long. The bullet gets embedded into the body and the unit (bullet + body) swings through some angle. Find out the angle through which the unit swings.

$m_1 = 2.5gm = 0.025$ kg $\qquad m_2 = 12kg$

$u_1 = 500m/sec$; $\qquad u_2 = 0$;

$v = commonvelocity\ of\ block\ and\ bullet\ after\ penetrate$;

Law of conservation of momentum



$$m_1u_1 + m_2u_2 = (m_1 + m_2) \times v$$

$$(0.025 \times 500) + (12 \times 0) = (0.025 + 12) \times v$$

$$v = \frac{12.5}{12.025} = 1.039\ m/sec$$

h- ht of the block move up after the bullet penetrate

$h = \frac{v^2}{2g} \qquad \therefore v = \sqrt{2gh}$

$$h = \frac{1.039^2}{2\ \times 9.81} = \mathbf{0.05502m}$$

$\theta - swing\ angle\ of\ block\ with\ bullet$

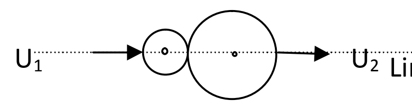
From diagram $h = 1 - 1\cos\theta$

$0.05502 = 1 - 1\cos\theta$; $\cos\theta = 1 - 0.05502$; $\qquad \theta = 19.09°$

10. A ball strikes centrally on another ball of mass twice the mass of first ball but moving with velocity 1/7 of the velocity of first ball and in the same direction. Show that, the first ball comes to rest after impact. The co-efficient of restitution between them 3/4.

$m_1 = mass\ of\ first\ ball$

$m_2 =\ mass\ of\ second\ ball = 2m$



$u_1 = velocity\ of\ first\ ball\ before\ impact$

$u_2 = velocity\ of\ second\ ball\ before\ impact = \frac{1}{7} \times u_1$

$e = \frac{3}{4} = 0.75$

Law of conservation of momentum

$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

$m_1u_1 + 2m_1 \times \frac{1}{7}u_1 = m_1v_1 + 2m_1v_2$

$\frac{7m_1u_1+2m_1u_1}{7} = m_1v_1 + 2m_1v_2$

$\frac{9}{7}m_1u_1 = m_1(v_1 + 2v_2\ )$

$u_1 = = \frac{7}{9}(v_1 + 2v_2\ )$……….(1)

Newton's law of collision

$(v_2 – v_1) = e\ (u_1 – u_2)$

$(v_2 – v_1) = (3/4)\ \{u_1 – (u_1/7)\}$

$(v_2 – v_1) = (3/4)(6/7)\ u_1$

$(v_2 – v_1) = (9/14)u_1$ …………………(2)

Solving 1 &2 we get $v_1 = 0$ i.e., first ball come to rest after impact.

Engineering Mechanics-Dynamics

Solutions for
University Question Paper-2012

UNIT – I

1. Two trains 'A' and 'B' leave the same station on parallel lines. Train 'A' starts with a uniform acceleration of 1/6 m/s² and attains the speed of 24 km/hr, then stem is reduced to keep the speed constant. Train 'B' leaves 40 seconds after, with uniform acceleration of 1/3 m/s² to attain the maximum speed of 48 km/hr. When will it overtake the train 'A'?

Consider the motion of train A

When uniform acceleration

$a = \frac{1}{6}\ {}^{m}/_{sec^2}$

$v = 24\ Km/\ hr$ = 6.67m/sec. $u = 0$

Eq. of motion $v = u + at$

$6.67 = 0 + \frac{1}{6} \times t$

Time for acceleration (t) = 40. Sec.

Distance travelled $s = ut + \frac{1}{2}at^2$

$= 0 + \frac{1}{2} \times \frac{1}{6} \times 40^2$

Distance travelled $s = 133.33m$

When uniform Velocity

$v = 6.67m/sec$ Let T – Total time taken by the train A while overtaken by B

Then time available for uniform velocity motion is T - 40.02

Eq. of motion $s = vt$

Distance travelled = 6.67× $(T - 40.02)$

Total distance travelled =133.33 + 6.67× $(T - 40)$

Consider the motion of train B

When uniform acceleration

$a = \frac{1}{3}\ {}^{m}/_{sec^2}$

$v = 48\ Km/\ hr$ = 13.3m/sec. $u = 0$

Eq. of motion $v = u + at$

$13.3 = \ 0 + \frac{1}{3} \times t$

Time for acceleration (t) = 40 Sec.

Distance travelled $s = ut + \frac{1}{2}at^2$

$= 0 + \frac{1}{2} \times \frac{1}{3} \times 40^2$

Distance travelled $s = 266.66m$

When uniform Velocity

$v = 13.3m/sec$

Time available for uniform Velocity of train B to over take the train A = Total time taken by A – time taken for acceleration of B - 40

= T -40-40 = T – 80 sec

Distance travelled $s = vt$

$= 13.3(T - 80)$

$= 13.3T - 1066.4$

Total distance travelled by car B = 266.66 + 13.3T-1066.4

$= 13.3T - 799.74$

Total distance travelled by car A = Total distance travelled by car B

$133.33 + 6.67(T - 40) = 13.3T - 799.74$

133.33 – 266.8 + 6.67 T = 13.3T -799.74

T = 100 s

$Time\ taken\ by\ car\ B\ to\ over\ take\ the\ car\ A = 100 - 40 = 60\ sec$

2. A stone dropped in to well and the sound of splash is heard in 6 sec. If the velocity of sound is 160 m/s, find the depth up to water level in the well. Take g = 9.1 m/s².

Consider motion of the stone

$u = 0, \quad s = dept\ of\ the\ well = h, \quad g = 9.81\ m/sec^{\ 2}$

$t - time\ taken\ by\ the\ stone$ to reach the water level

$$s\ = ut + \frac{1}{2}gt^2$$

$$h = \frac{1}{2} \times 9.81\ t^2$$

$h\ = 4.905\ t^2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots..(1)$

Consider motion of the sound

$s = h = \ depth\ of\ well; \ time\ taken\ by\ sound\ 6 - t; \ v = 160m/sec \Rightarrow$

Eq. of motion

$$s = vt$$

$$h = 160 \times (6 - t)$$

$$h = 960 - 160t - - - - \rightarrow (2)$$

Equate (1)&(2)

$$4.905 + 160t - 960 = 0$$

$$t = \frac{-160 \pm \sqrt{160^2 - 4 \times 4.905 \times 960}}{2 \times 4.905}$$

$$t = \frac{-160 \pm 210}{2 \times 4.99.8105}$$

$$= 5.096\ sec$$

$depth\ of\ well\ l \qquad = 4.905 \times 5.096^2 \qquad = 127.37m.$

UNIT - II

3. Two balls are projected from the same point in directions inclined at 60º and 30º to the horizontal. If they attain the same maximum height, what is the ratio of their velocities of projection?

H is the max height attained by the two projectiles

Max ht for 60º projectile. $H = \frac{u^2 sin^2 \propto}{2g}$

$$H = \frac{{u_1}^2 sin^2 60}{2 \times g} \text{ ------------→ } (1)$$

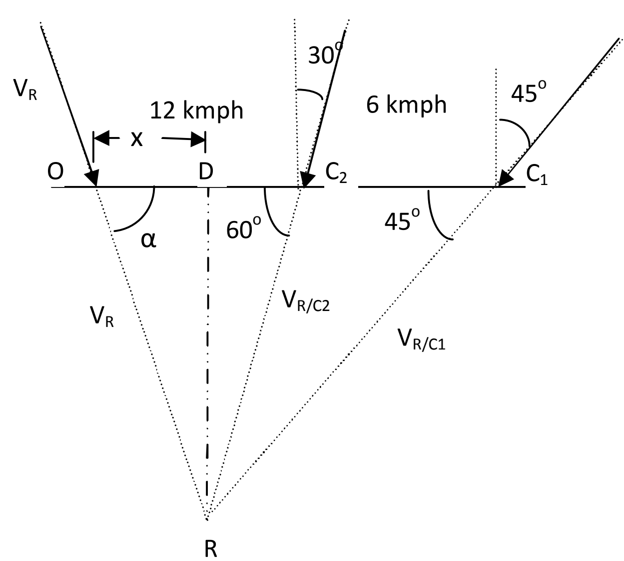
Max ht for 30º projectile. $\quad H = \frac{{u_1}^2 sin^2 30}{2 \times g}$ ------------→ (2)

Equate (1)&(2)

$$\frac{{u_1}^2 sin^2 60}{2g} = {u_2}^2 sin^2 30$$

Ratio of the velocity $\frac{u_1}{u_2} = \sqrt{\frac{sin^2 30}{sin^2 60}} = \frac{\sin 30}{\sin 60} =$ 0.5773

4. When a cyclist is riding towards the East at 18 km/hr, he finds the rain at an angle of 45⁰with the vertical. When he rides at 12 km/hr, he meets the rain at an angle of 30⁰ with the vertical. What is the actual velocity in magnitude direction of rain?



Take triangle c_2DR

$\tan 60° = \frac{RD}{Dc_2}$; RD= $Dc_2 \sin 60° = (12 - x) \tan 60$ ------------------------ $\rightarrow (1)$

Take triangle C_1DR

$\tan 45° = \frac{RD}{DC_1}$; RD = $DC_1 \tan 45 = (18 - x) \tan 45$ $------\rightarrow (2)$

Equate (1)&(2)

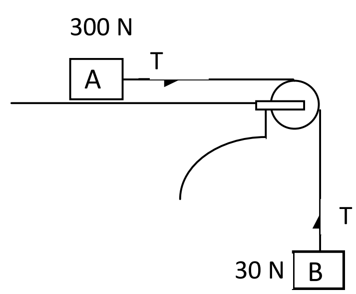
$$(12 - x) \tan 60 = (18 - x) \tan 45$$
$$(12 - x) \frac{\tan 60}{\tan 45} = (18 - x)$$
$$(12 - x) 1.732 = (18 - x)$$
$$0.732x = 2.784$$
$$x = 3.804\ km/hr$$
$$\therefore RD = (12 - x) \tan 60 \quad = (12 - 3.804) \tan 60$$
$$= 14.195$$

$$Velocity\ of\ rain\ V_R = \sqrt{RD^2 + x^2} = \sqrt{14.195^2 + 3.804^2}$$
$$V_R = 14.69 km/hr$$
$$\propto = \tan^{-1} \frac{RD}{x}$$
$$= \tan^{-1} \left(\frac{14.195}{3.804}\right) = 75°$$

Angle of rain with vertical = 90 - 75 = 15°

UNIT – III

5. The following figure shows of a body of weight 300 N on a smooth horizontal plane is attached by a string to a 30 N weight, which hangs vertically. Find the acceleration of the system and tension in the string.



<u>Consider 300N Block</u>

Smooth surface therefore no friction

By D' Alembert Principle

$$\sum F = m \times a$$
$$T = m \times a\ ;$$
$$T = \frac{300}{9.81} a$$
$$T = 30.86a \text{---------------------} \rightarrow (1)$$

Consider 30N Block
Smooth surface therefore no friction

By D' Alembert Principle

$\sum F = m \times a$

$W - T = m \times a$

$30 - T = \frac{30}{9.81} \times a$

$30 - T = 3.058 \times a$ ————————————→ (2)

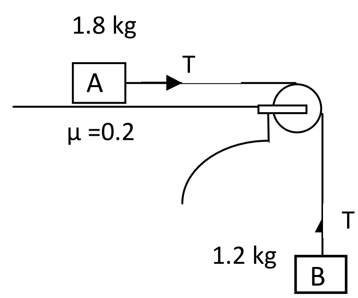
Add (1)&(2) $\Rightarrow (30.58 + 3.058)a = 30$

$a = 0.891 m/sec^2$

Apply 'a' value in eq ----------(1)

$T = 30.58 \times 0.891$; T =31.47N

6. Two masses of 1.8 kg and 1.2 kg are connected by an inextensible string, passing over a frictionless pulley. Calculate: (i) Acceleration of the system, when released from rest. (ii) Tension in the sting. Take μ = 0.2.



Consider 18 kg mass
Frictional resistance

$F_r = \mu \times R_N = \mu \times w$
$= 0.2 \times 1.8 \times 9.81 = 3.53N$

By D' Alembert Principle

$\sum F = m \times a$

$T - F_r = m \times a$

$T - 3.53 = 1.8 \times a$ ————————————→ (1)

Consider 12 kg mass

By D' Alembert Principle

$\sum F = m \times a$

$w - T = m \times a$

$1.2 \times 9.81 - T = 1.2 \times a$

$11.77 - T = 1.2 \times a$ ————————————→ (2)

Add (1)&(2)

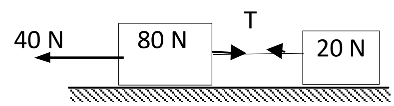
$8.24 = 3a$

$a = 2.746\ m/sec^2$

Apply 'a' value in eq ----------(1)

$T = 1.8 \times 2.746 + 3.53$ = 8.47N

7. Two weights 80 N and 20 N are connected by a thread and move along a rough horizontal plane under the action of a force 40 N, applied in the first weight of 80 N as shown below. The co-efficient of friction between the sliding surfaces of the weights and the plane is 0.3. Determine the acceleration of the weights and the tension in the thread using work – energy equation.



.<u>Consider 20N Block</u>

Frictional resistance

$$F_r = \mu \times R_N = \mu \times w$$
$$= \quad 0.3\times 20 = 6N$$

Using Work–energy equation

$$\sum F \times s = \frac{1}{2}m({v_2}^2 - {v_1}^2)$$

$$(T - F_r) \times s = \frac{1}{2}m({v_2}^2 - {v_1}^2)$$

$$(T - 6) \times s = \frac{1}{2} \times \frac{20}{9.81}({v_2}^2 - {v_1}^2)$$

$$(T - 6) \times s = 1.019({v_2}^2 - {v_1}^2) - - - - - \rightarrow (1)$$

<u>Consider 80N Block</u>

Frictional resistance $F_r = \mu \times R_N = \mu \times w$

$= \quad 0.3\times 80 = 24N$

Force applied F= 40N

Using Work – energy equation

$$\sum F \times s = \frac{1}{2}m({v_2}^2 - {v_1}^2)$$

$$(F - T - F_r) \times s = \frac{1}{2}m({v_2}^2 - {v_1}^2)$$

$$(40 - T - 24) \times s = \frac{1}{2} \times \frac{20}{9.81}({v_2}^2 - {v_1}^2)$$

$$(16 - T) \times s = 4.077({v_2}^2 - {v_1}^2) - - - - - \rightarrow (2)$$

$$\frac{(2)}{(1)} \Rightarrow \frac{(16-T)\times s}{(T-6)\times s} = \frac{4.077({v_2}^2-{v_1}^2)}{1.019({v_2}^2-{v_1}^2)}$$

$$(16 - T) = 4\,(T - 6)$$

$$(16 - T) = 4\,T - 24$$

$$5\,T = 40; \quad T = 8\,N$$

Apply T value in eq (1)

$$\big((8 - 6)\big)s = 1.019({v_2}^2 - {v_1}^2)$$

$2\ s = 1.019({v_2}^2 - {v_1}^2)$
${v_2}^2 - {v_1}^2 = 1.962\ s$
${v_2}^2 - {v_1}^2 = 2as \ \Rightarrow \qquad a = \frac{{v_2}^2 - {v_1}^2}{s}$
Equation of motion
${v_2}^2 - {v_1}^2 = 2as$
$1.962 \times S = 2 \times a \times S$
$a = 0.981 m/s^2$

8. A ball of mass 'm' is dropped on to a spring of stiffness 'K' from height 'h'. Find the maximum deflection of the spring. Assume, m = 5 kg; K =500 N/ m; h = 100 mm.

$h = 100m$; Let x Deflection of the spring
Work –Energy relationship

W.D by ball + W.D by spring = Change in kinetic Energy
$m * g(h + x) - \frac{1}{2}kx^2 = 0$
$5 \times 9.81(0.1 + x) - \frac{1}{2} \times 500 \times x^2 = 0$
$49.05 \times 0.1 + 49.05x - 250x^2 = 0$
$250x^2 - 49.05x - 4.905 = 0$
$x = \frac{49.05 \pm \sqrt{49.05^2 + 4 \times 250 \times 4.905}}{2 \times 250a}$
$= \frac{49.05 \pm 85.5}{2 \times 250}$

$= 0.2691m \qquad Deflection\ the\ spring = 269.1mm$

UNIT – V

9. A pile having a weight of 5000 N is driven into the ground by dropping a hammer of weight 3180 N at a height of 2.7 m. The pile is driven into the ground by 0.15. Calculate the average resistance of the soil.

Weight of hammer'W'=3180N
Weight of pile w =5000N
Average resitance R=?
V- common velocity of pipe and hammer
Law of conservation of momentum
$M \times u_H + m \times u_P = (M + m)v$

$(\frac{3180}{g}) \times \sqrt{2 \times g \times 2.7} = \frac{(5000 + 3180)}{g} v$

$v = \frac{23118.6}{8180} = 2.826 m/s$

Applying work energy principle

$$\{-R + (M+m)g\}x = \frac{1}{2}(M+m)(0 - v^2)$$

$$\{-R + 8180\} \times 0.15 = -\frac{\frac{1}{2}(3180+5000)}{g} \times 2.826^2$$

-R + 8180 = -22197.66

R = 14017 N

10. A ball is dropped from a height of 10 m on a fixed steel platform. Determine the height to which the ball rebounds on the first, second and third bounces. The co-efficient of restitution between the ball and the plate is 0.9.

$h_0 = 10m; \quad e = 0.9$

For first balance

$u - velocity\ of\ ball\ before\ first\ Bounced.$

$= \sqrt{2gh_0} = 2 \times 9.81 \times 10 = 14.007\ m/s$

$v - velocity\ of\ ball\ After\ first\ Bounced..$

$= \sqrt{2gh_1}$ $\qquad h_1 - h_t\ reach\ after\ first\ Bounced$

We know that $e = \frac{v}{u}$

$$0.9 = \frac{\sqrt{2gh1}}{14.007}$$

$h1 = \frac{14.007 \times 0.9}{\sqrt{2g}}$= 8.1m

<u>For second *Bounce*</u>

$u = \sqrt{2gh_1} = \sqrt{2g} \times \sqrt{8.1}$

$v = \sqrt{2gh_2}$

$$e = \frac{v}{u}; \quad 0.9 = \frac{\sqrt{2g} \times \sqrt{h_2}}{\sqrt{2g} \times \sqrt{8.1}}$$

$h_2 = 0.9^2 \times 8.1 = 6.561m$

<u>For third *Bounce*</u>

$u = \sqrt{2gh_2} = \sqrt{2g} \times \sqrt{6.561}$

$v = \sqrt{2gh_3}$

$$e = \frac{v}{u}; \quad 0.9 = \frac{\sqrt{2g} \times \sqrt{h_3}}{\sqrt{2g} \times \sqrt{6.561}}$$

$h_3 = 0.9^2 \times 6.561 = 5.314m$

Engineering Mechanics-Dynamics

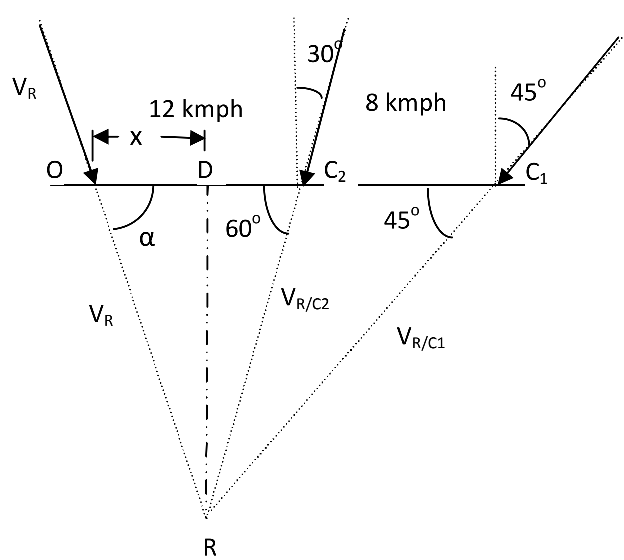
Solutions for
University Question Paper-2013

UNIT – I

1. A bus is beginning to move with an acceleration of 0.3 m/s². A man who is 12 m behind the bus starts running at 3 m/s to catch the bus. After how many seconds will the man be able to catch the bus?

.Bus (start from rest)
$a = 0.3m/sec^2$ $u = 0$ t=Bus time(The man catch the bus)
$s_B = Distance\ moved\ by\ the\ bus$
$s = ut + \frac{1}{2}at^2$
$s = 0 + \frac{1}{2} \times 0.3 \times t^2$
$s_B = 0.15\ t^2$ ……………………….(1)
Man(running at uniform velocity)
$v = 3m/s$
$t = time\ for\ the\ man\ catch\ the\ bus$
$s = v \times t$
$s_M = 3t$ ———————————→ (2)
$But\ \ s_m = s_B + 12$
Apply (2) in (1)
$3t = 0.15\ t^2 + 12$
$0.15t^2 - 3t + 12 = 0$
$$t = \frac{3 \pm \sqrt{9 - 4 \times 0.15 \times 12}}{2 \times 0.15}$$
$t = 14.47$ sec or 5.53 sec

2. *When a cyclist is riding West at 20 km/hr, he finds the rain meeting him an at angle of 45⁰ with vertical; when he rides at 12 km/hr, he meets the rain at an angle of 30⁰ with the vertical. What is the actual velocity (magnitude and direction) of the rain?*



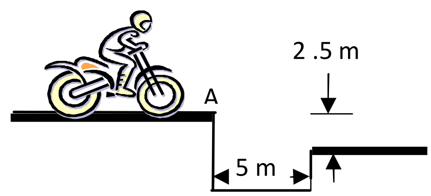
Let the distance OD = x
OC_1- Velocity of cycle=20 Kmph
OC_2- Velocity of cycle=12 Kmph
From the triangle DC_1R
$$\tan 45° = \frac{RD}{C_1D} = \frac{RD}{20 - x}$$

$\therefore RD = \tan 60\,(12 - x) ----------\rightarrow (2)$
From eqn(1)&(2)
$\tan 45°\,(20 - x) = \tan 60°\,(12 - x)$
$x = 1.072 Kmph$
RD=18.92
Velocity of rain $V_R = \sqrt{x^2 + RD^2} = \sqrt{1.072^2 + 18.92^2}$
$V_R = 18.95 Kmph$
$\propto = \tan^{-1}\left(\frac{RD}{x}\right) = \tan^{-1}\frac{18.92}{1.072} = 86.75°$
Direction of velocity of rain with respect to vertical=90-86.75° = 3.24°

UNIT – II

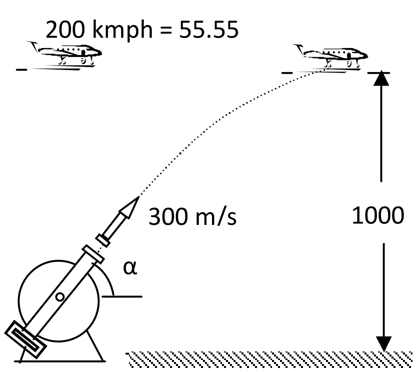
3. A motor cyclist wants to jump over a ditch which is 5 m wide and the other bank of which is lower by 2.5 m from the bank, he intends to starts the jump. Calculate the minimum velocity so that he can accomplish the task. With what velocity he will reach the other bank?

Downward motion of motor cycle
The motor cycle move in Horizontal direction, the vertical component of it velocity is Zero
$u_y = 0;$ $a_y = 9.81 m/sec^2$ $S_y = 2.5m$
$s = ut + \frac{1}{2}at^2$
$2.5 = 0 + \frac{1}{2} \times 0.3 \times t^2$
$t = \sqrt{\frac{2.5 \times 2}{9.81}}$
$t = 0.7139\, sec.$



Vertical component of the velocity of the motor cycle after clearing the ditch.
$v_y = u_y + at$
$v_y = 0 + 9.81 \times 0.7139$
$v_y = 7 m/sec$
Horizontal motion of the motor cycle
$s_x = v_x \times t$ $s_x = 5m$
$5 = v_x \times 0.7139$
$v_x = 7 m/sec.$(Minimum velocity of the notorcycle)
Velocity of after clearing the ditch

$v = \sqrt{v_x{}^2 + v_y{}^2}$ $v = \sqrt{7^2 + 7^2}$ = 9.89 m/sec



4. An airplane is flying on a straight level course at 200 km/ hr at a height of 1000 m above the ground. An air craft gun located on the ground fires a shell with an initial velocity above it. At what inclination, to the horizontal, should the gun be fired to hit the plane? What will then be the horizontal distance of the plane from the gun?

Plane horizontal velocity = 200kmph = 55.55 m/sec
Velocity of the shell = 300 m/ sec
Inclination of shell = $\propto$
Vertical motion of the shell
$u_y = 300 \sin \propto$; $a = -9.81 m/sec$; t- time taken to hit plane $s_y = 1000m$

$$s = ut + \frac{1}{2}at^2$$

$1000 = 300 \times t + \frac{1}{2} \times 9.81 \times t^2 ------\rightarrow$ ①(1)

Horizontal motion of the shell
$u_x = u \cos \propto = 300 \cos \propto$; $s_x -$ *horizontal distance moved by the plane*
$s = u_x \times t = 300 \cos \propto \times t ---------\rightarrow$ ②(2)

Horizontal motion of the plane
Distance moved $s = v \times t = 55.55 \times t ----\rightarrow$ ③(3)
Equate ②&③ 2 & 3
$55.55 \times t = 300 \cos \propto \times t$

$$\cos \propto = \frac{55.55}{300} \quad \therefore \propto = 79.33°$$

Substitute the value of $\propto$ in eqation ① (1) we get
$1000 = 300 \sin 79.33 \times t + 4.905 \times t^2$
$4.905t^2 - 294.81t + 1000 = 0$

$$t = \frac{294.81 \pm \sqrt{294.81^2 - 4 \times 4.905 \times 1000}}{2 \times 4.905}$$

$t = 3.608\, sec$
Apply 't' value in eq ③
$s = 55.55 \times 3.608$
$s = 200.42m$

UNIT – III

5. A train of weight 2,000 kN starts from rest and attains a speed of 100 km/hr in 240 seconds. The frictional resistance of the track is 8 N/kN of the weight of the train. Calculate the pull applied.

$W = 200KN$; $u = 0$; $v = 100km/hr = 27.77m/\text{sec}$; $t = 240\, sec$;

$$Frctional\ resitance\ F_r = \frac{8N}{KN} = 8 \times 2000 = 16000\ N$$

Equation of Motion
$v = u + at$
$27.77 = 0 + a \times 240$
$a = 0.115 m/sec^2$
Applying D' Alembert Principle

$$\sum F = m \times a$$

$F_T - F_r = m \times a$

$$F_T - 16000 = \frac{2000 \times 1000}{9.81} \times 0.115$$

F_T =39.445KN   ∴ *Pull applied on the train* =39.445 kN

6. A body weighing 300 N pushed up at 30⁰ plane by a 400 N force acting parallel to the plane. If the initial velocity of the body is 1.5 m/s and co-efficient of kinetic friction is 0.2, what velocity will the body have after moving 6 m?

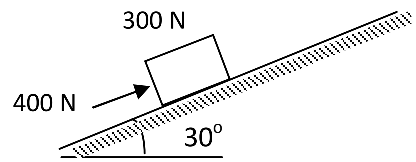
$W = 300\ kN$; $u = 1.5m/\text{sec}$ $\mu = 0.2$; $\propto = 30°$; $s = 6m$; $F_t = 400N$

The body moving up

Frictional force $F_r = \mu \times R_N$

$= \mu \times w \cos \propto$

$= 0.2 \times 300 \times \cos 30$ =51.96 N



Applying D' Alembert Principle

$$\sum F = m \times a$$

$$F_T - w \sin \alpha - 51.96 = \frac{300}{9.81} \times a$$

400- 300 sin30 – 51.96 = 30.58 a

$a = 6.47 m/sec^2$

Equation of Motion

$v^2 - u^2 = 2as$

$v^2 - 1.5^2 = 2 \times 6.47 \times 6$

$v = 8.93 m/sec^2$ $\boldsymbol{velocity\ of\ thebody} = \mathbf{8.93} m/sec^2$

UNIT – IV

7. A body of weight 60 N is projected up a 15⁰ inclined plane with a velocity of 10 m/s. The co-efficient of kinetic friction between the block and the plane is 0.2. Find the maximum distance that the body will move up the plane before it comes to rest.

$W = 60\ kN$; $\mu = 0.2$; $\alpha = 15°$; $v_1 = 10m/\sec$; $s = ?$

Frictional force $F_r = \mu \times R_N$

$= \mu \times w \cos 15 = 0.2 \times 60 \cos 15$= 11.59 N

Work –Energy relationship

$$\sum F \times s = \frac{1}{2} m({v_2}^2 - {v_1}^2)$$

$$(-w \sin \propto - F_r) \times s = \frac{1}{2} m({v_2}^2 - {v_1}^2)$$

$$(-60 \sin 15 - 11.59) \times s = \frac{1}{2} \times \frac{60}{9.81} \times (0 - 10^2)$$

$s = 11.27m$

8. A car of mass 300 kg is travelling at 36 km/hr. on a level road. It is brought to rest, after travelling a distance of 5 m. What is the average force of resistance acting on the car?

$m = 300kg$; $u_1 = 36km/hr$; $u_2 = 0$; $s = 5m$:

Work –Energy relationship

$$\sum F \times s = \frac{1}{2} m({v_2}^2 - {v_1}^2)$$

$$-F_r \times 5 = \frac{1}{2} \times 300(0 - 10^2)$$

$F_r = 300N$

Force of resistance acting on the car=300N

$$-F_r \times s = \frac{1}{2} m({v_2}^2 - {v_1}^2)$$

9. A pile having a weight of 5,000N is driven into the ground by dropping the hammer of weight 3180 N at a height of 2.7 m. The pile is driven into the ground by 0.15 m. Calculate the average resistance of the soil.

$W_1 = 3180N$; $W_2 = 5000N$; $W = 9180N$; $l = 2.7m$; $s = 0.15m$

Velocity of hammer before impact $u_1 = \sqrt{2gh}$

$$= \sqrt{2 \times 9.81 \times 2.7}$$
$$= 7.27 m/sec.$$

Total momentum before impact = Total momentum After impact

$\therefore v - common\ velocity\ of\ hammer\ and\ pile$

$$u_1 m_1 + u_2 m_2 = (m_1 + m_2) \times v$$

$$7.27 \times \frac{3180}{g} + 0 = \left(\frac{3180 + 5000}{g}\right) \times v$$

$v = 2.826 m/sec$

Work done by soil resistance+ change in kinetic energy.

$$(-R + W)S = \frac{1}{2} m({v_2}^2 - {v_1}^2)$$

$$(-R + 8180) \times 0.15 = \frac{1}{2} \times \frac{8180}{9.81} (0 - 2.826^2)$$

Resistance by soil =14017 N

10. Two bodies, one of mass 30 kg, moves with a velocity of 9 m/s strikes on another body of mass 15 kg, moving in the opposite direction with the velocity of 9 m/s centrally. Find the velocity of each body after impact, if the co-efficient of restitution is 0.8.

$m_1 = 30\ kg$ $m_2 = 15 kg$

$u_1 = 9\ m/sec$ $u_2 = 9 m/sec$

$e = 0.8$

Direct central impact

Law of conservation of momentum principle

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$
$$30 \times 9 + (-15 \times 9) = 30 \times v_1 + 15 \times v_2$$
$$30\ v_1 + 15 v_2 = 135$$

$2v_1 + v_2 = 9 ------\!\!\longrightarrow$ (1)Newton's law of collision.

$$(v_2 - v_1) = e(u_1 - u_2)$$
$$(v_2 - v_1) = 0.8[9 - (-9)]$$
$$(v_2 - v_1) = 14.4 -------\!\!\longrightarrow (2)$$
$$(1) + (2) \times 30 \cdots\cdots \Rightarrow 45 v_2 = 135 + (30 \times 14.4)$$
$$45 v_2 = 567$$
$$v_2 = 12.6 m/sec$$

Apply v_2 value in eq (2)

12.6 - $v_1 = 14.4$

$\therefore v_1 = -1.8 m/sec.$